Sources of Mechanical Energy

- Potential Energy

$$E = mgh$$

- Kinetic Energy

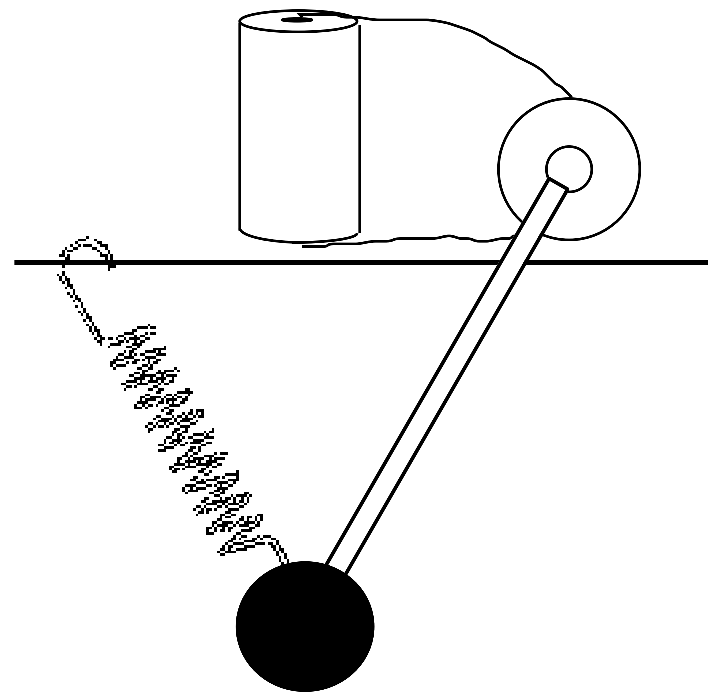
$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

- Spring Energy

$$E = \frac{1}{2}kx^2$$

- Electrical Energy + Motor

$$E = VIt = \tau\omega t$$



Batteries

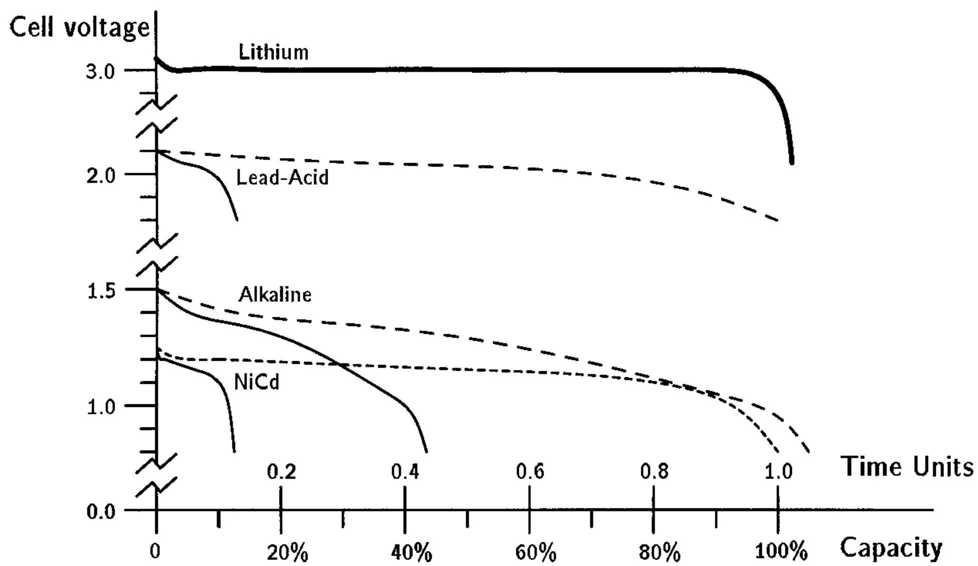
- Types of Batteries
  - Alkaline
  - Ni-Cd
  - NiMH
  - Lead-Acid
  - Lithium
- Battery Properties
  - Rechargeability
  - Energy Density
  - Capacity
  - Voltage
  - Internal Resistance
  - Discharge Rate
  - Shelf Life

Battery Chemistry	Recharge	Energy Density (Whr/kg)	Cell Voltage	Typical Capacity (mAh)	Internal Resistance (ohms)	Comments
Alkaline	No	130	1.5	AA 1400 C 4500 D 10000	0.1	Most common primary battery
Lead-Acid	Yes	40	2.0	1.2 - 120 Ah	C-size 0.006	Available in a wide variety of sizes
Lithium	No	300	3.0	A 1800 C 5000 D 14000	0.3	Execellent energy density, high unit cost
Mercury	No	120	1.35	Coin 190	10	
NiCd	Yes	38	1.2	AA 500 C 1800 D 4000	0.009	Low internal resistance, available from many sources
NiMH	Yes	57	1.3	AA 1100 4/3A 2300		Better energy density than NiCd, expensive
Silver	No	130	1.6	Coin 180	10	
Zinc-Air	No	310	1.4			High energy density but not widly available, limited range of sizes
Carbon-Zinc	No	75	1.5	D 6000		Inexpensive but obsolete

All numbers listed here are approximate. Precise values depend on the details of the particular battery. Some values depend on the battery's state of charge, temperature, and discharge history.

Figure 8.1: Comparison of characteristics for selected batteries and sizes.

Battery Discharge Curves



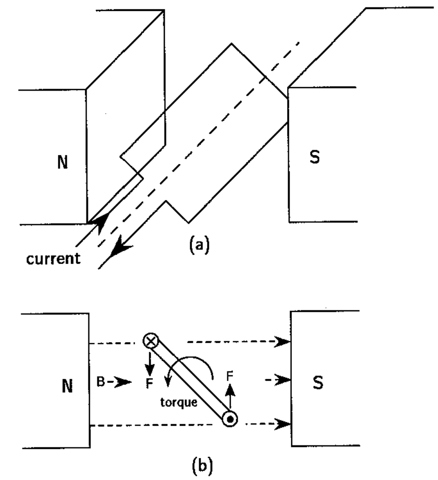
- The graph is normalized with respect to a lithium battery
- The dashed lines show output voltage versus battery capacity consumed
- The solid lines show voltage versus time

DC Motor Model

Physical Principles

- Current through a wire produces a magnetic field
- A wire moving through a magnetic field induces a current
- For multiple windings we find that

$$\tau = K_t i$$

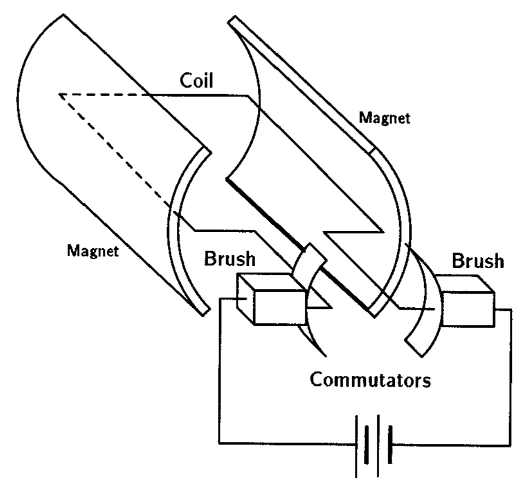


Motor electrical model

$$L_a \frac{di_a}{dt} + R_m i + K_e \omega = V_a$$

Substituting in for i and solving for τ gives

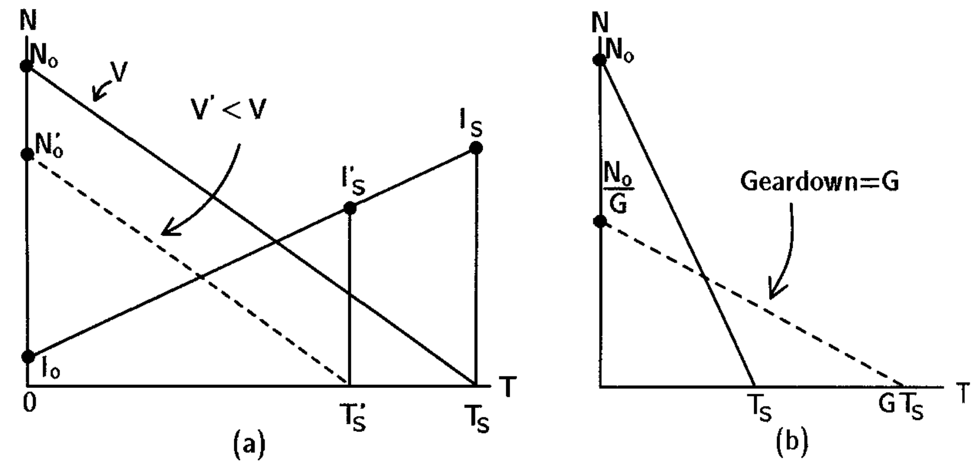
$$\tau = \frac{K_t}{R_m} V_a - \frac{K_t K_e}{R_m} \omega$$



Torque-Speed Curves

- DC Motor Equation

$$\tau = \frac{K_t}{R_m} V_a - \frac{K_t K_e}{R_m} \omega$$



- At $\omega = 0$

$$\tau = \frac{K_t}{R_m} V_a = \tau_{stall}$$

- At $\tau = 0$

$$\omega = \frac{V_a}{K_e} = \omega_{noload}$$

Torque-Speed Equation

$$\tau = \tau_{stall} - \frac{\tau_{stall}}{\omega_{noload}} \omega$$

A transmission changes the slope of the torque-speed curve (line) to provide more desirable no load speed and output torque characteristics.

Motor Power

When does a motor operate at maximum power?

$$P = \tau\omega = (\tau_{stall} - \frac{\tau_{stall}}{\omega_{noload}}\omega)\omega$$

To find the maximum operating point, take the derivative and set it equal to zero.

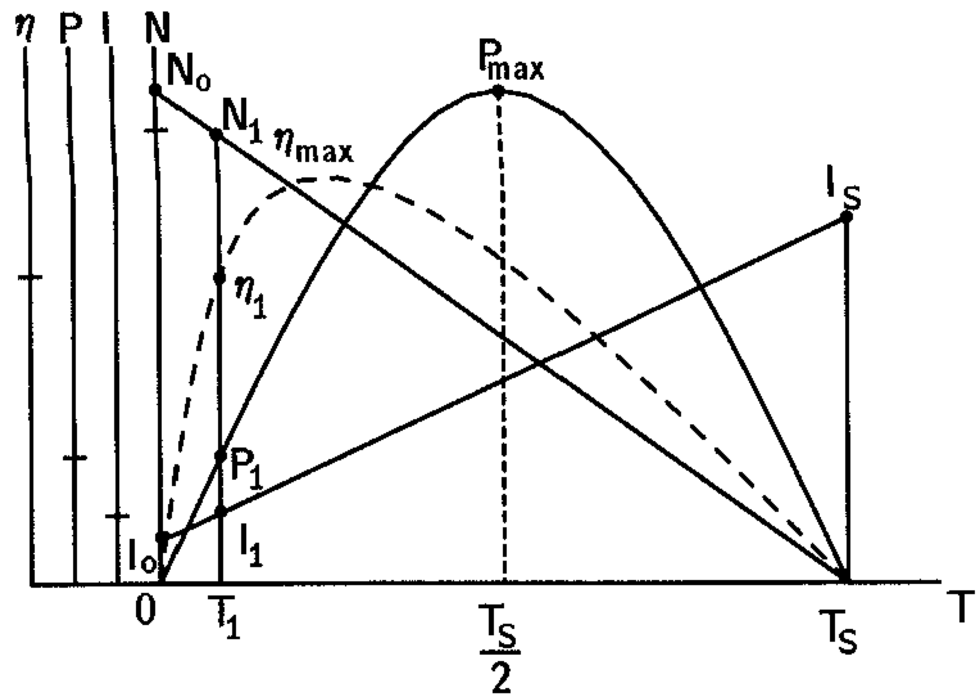
$$\frac{\partial P}{\partial \omega} = \tau_{stall} - \frac{2\tau_{stall}}{\omega_{noload}}\omega = 0$$

Solve for ω^* and corresponding τ^*

$$\omega^* = \frac{\omega_{noload}}{2}$$

$$\tau^* = \tau_{stall} - \frac{\tau_{stall}}{\omega_{noload}}\frac{\omega_{noload}}{2} = \frac{\tau_{stall}}{2}$$

Power and Efficiency



$$I_{stall} = \frac{V_{in}}{R_m}$$

Efficiency: $$\eta_{max} = \left(1 - \sqrt{\frac{I_{noload}}{I_{stall}}}\right)$$